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PARALLEL OPTIMIZATION METHODS FOR ESTIMATION

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Outline

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Research Overview

- Parallel Methods for Nonsmooth and Nonconvex Optimization
- Array Signal Processing: Sparse Direction-of-Arrival (DOA) Estimation Methods
- Optimization and Learning over Graphs
- Game Theory: Distributed Generalized Nash Equilibrium Seeking

Smoothing Successive Convex Approximation

- Smothing Majorization + Successive Convex Approximation (SCA)
- Convergence analysis

Application to Phase Retrieval[a]

[a] Tianyi Liu et al. “Extended Successive Convex Approximation for Phase Retrieval with Dictionary Learning”. In: *IEEE Trans. Signal Process.* 70 (2022), pp. 6300–6315. ISSN: 1941-0476. DOI: 10.1109/TSP.2022.3233253.

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Grid-based Sparse DOA Estimation[a]

- $\ell_{2,0}$-norm constrained grid-based formulation
- Mixed-integer semidefinite program (MISDP) reformulation

Gridless DOA Estimation in Distributed Arrays[b]

- Distributed arrays with inter-subarray phase/displacement uncertainties
- Gridless relaxation of $\ell_{2,1}$-norm minimization based on shift invariances
- Parallel algorithm based on ADMM and SCA

[a] Tianyi Liu et al. *Maximum a posteriori direction-of-arrival estimation via mixed-integer semidefinite programming*. Oct. 2024. DOI: 10.48550/arXiv.2311.03501. arXiv: 2311.03501.

[b] Tianyi Liu et al. *Gridless Parameter Estimation in Partly Calibrated Rectangular Arrays*. June 2024. DOI: 10.48550/arXiv.2406.16041. arXiv: 2406.16041 [eess].

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Graph Topology Learning[a]

- Learning sparse Gaussian graphical model with Laplacian constraints
- Block SCA algorithm
- Evaluation on real-world data, including meteorological, traffic, financial...

[a] Tianyi Liu et al. "A Block Coordinate Descent Algorithm for Sparse Gaussian Graphical Model Inference with Laplacian Constraints". In: *IEEE Int. Workshop Comput. Adv. Multi-Sens. Adapt. Process.* Dec. 2019, pp. 236–240. DOI: 10.1109/CAMSAP45676.2019.9022643.

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Phase Retrieval

In classic phase retrieval problem, estimate $\boldsymbol{X}$ from magnitude of linear measurements:

$$\boldsymbol{Y} = |\mathcal{F}(\boldsymbol{X})| + \boldsymbol{N}$$

- $\boldsymbol{X} \in \mathbb{C}^{N \times I}$ is the signal to estimate
- $\mathcal{F} : \mathbb{C}^{N \times I} \to \mathbb{C}^{M_1 \times M_2}$ is the linear measurement operator
- $\boldsymbol{N} \in \mathbb{R}^{M_1 \times M_2}$ is a matrix of i.i.d. noise entries
- $\boldsymbol{Y} \in \mathbb{R}_+^{M_1 \times M_2}$ is the magnitude-only measurements
- Absolute value operation performed elementwise

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Phase Retrieval with Dictionary Learning

- Additional prior information of signal $\boldsymbol{X}$, e.g., sparsity
- Original signal $\boldsymbol{X} = [\boldsymbol{x}_1, \ldots, \boldsymbol{x}_I]$ admits a sparse representation:

$$\boldsymbol{x}_i = \boldsymbol{D}\boldsymbol{z}_i$$

  - $\boldsymbol{D} = [\boldsymbol{d}_1, \ldots, \boldsymbol{d}_P] \in \mathbb{C}^{N \times P}$ is an unknown dictionary
  - $\boldsymbol{Z} = [\boldsymbol{z}_1, \ldots, \boldsymbol{z}_I] \in \mathbb{C}^{P \times I}$ is a sparse matrix of codes

Problem statement

Given magnitude-only measurements $\boldsymbol{Y}$ and linear operator $\mathcal{F}$, jointly learn an unknown dictionary $\boldsymbol{D}$ and a sparse code matrix $\boldsymbol{Z}$ such that $\boldsymbol{Y} \approx |\mathcal{F}(\boldsymbol{D}\boldsymbol{Z})|$

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Phase Retrieval with Dictionary Learning
Problem Formulation

- Proposed compact formulation:

$$\min_{\boldsymbol{D} \in \mathcal{D}, \boldsymbol{Z}} \quad \underbrace{\frac{1}{2}\|\boldsymbol{Y} - |\mathcal{F}(\boldsymbol{DZ})|\|_F^2}_{\text{loss}} + \underbrace{\lambda\|\boldsymbol{Z}\|_{1,1}}_{\text{sparsity}}$$

  - $\|\boldsymbol{Z}\|_{1,1} = \sum_{p=1}^{P}\sum_{i=1}^{I}|z_{pi}|$
  - $\lambda > 0$: sparsity regularization parameter
  - To avoid scaling ambiguities, restrict $\boldsymbol{D} \in \mathcal{D} = \{\boldsymbol{D} \mid \|\boldsymbol{d}_p\|_2 \leq 1, p = 1, \ldots, P\}$
  - Loss function is nonsmooth and nonconvex

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Phase Retrieval with Dictionary Learning
Problem Formulation

- Proposed compact formulation:

$$\min_{\boldsymbol{D}\in\mathcal{D},\boldsymbol{Z}} \quad \underbrace{\frac{1}{2}\|\boldsymbol{Y}-|\mathcal{F}(\boldsymbol{DZ})|\|_F^2}_{\text{loss}} + \underbrace{\lambda\|\boldsymbol{Z}\|_{1,1}}_{\text{sparsity}}$$

- Alternative formulation [QP17][1]:

$$\min_{\boldsymbol{X},\boldsymbol{D}\in\mathcal{D},\boldsymbol{Z}} \quad \underbrace{\frac{1}{2}\|\boldsymbol{Y}-|\mathcal{F}(\boldsymbol{X})|\|_F^2}_{\text{data fitting}} + \underbrace{\frac{\mu}{2}\|\boldsymbol{X}-\boldsymbol{DZ}\|_F^2}_{\text{approximation quality}} + \underbrace{\rho\|\boldsymbol{Z}\|_{1,1}}_{\text{sparsity}}$$

  - Nonconvex and nonsmooth
  - Additional auxiliary variable $\boldsymbol{X}$ and two regularization parameters

[1] Tianyu Qiu and Daniel P. Palomar. “Undersampled Sparse Phase Retrieval via Majorization–Minimization”. In: *IEEE Trans. Signal Process.* 65.22 (Nov. 2017), pp. 5957–5969. ISSN: 1053-587X. DOI: 10.1109/TSP.2017.2745459.

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Majorization-Minimization

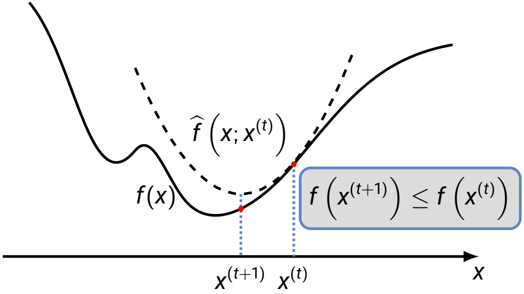
$$\min_{\boldsymbol{D}\in\mathcal{D},\boldsymbol{Z}} \underbrace{\frac{1}{2}\|\boldsymbol{Y}-|\mathcal{F}(\boldsymbol{DZ})|\|_F^2}+\lambda\|\boldsymbol{Z}\|_{1,1}$$

↓ Majorization (upper bound)

$$\frac{1}{2}\left\|\boldsymbol{Y}^{(t)}-\mathcal{F}(\boldsymbol{DZ})\right\|_F^2$$

- $\boldsymbol{Y}^{(t)}$: Magnitude measurements $\boldsymbol{Y}$ with phases of $\mathcal{F}(\boldsymbol{D}^{(t)}\boldsymbol{Z}^{(t)})$

Classic Majorization-Minimization (MM)



- Differential consistency:

$$\hat{f}'_{\boldsymbol{d}}(\boldsymbol{x}^{(t)};\boldsymbol{x}^{(t)})=f'_{\boldsymbol{d}}(\boldsymbol{x}^{(t)})\ \forall\boldsymbol{d}$$

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Majorization-Minimization

$$\min_{\boldsymbol{D} \in \mathcal{D}, \boldsymbol{Z}} \underbrace{\frac{1}{2}\|\boldsymbol{Y} - |\mathcal{F}(\boldsymbol{D}\boldsymbol{Z})|\|_{\mathrm{F}}^2}_{} + \lambda \|\boldsymbol{Z}\|_{1,1}$$

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$$\frac{1}{2}\left\|\boldsymbol{Y}^{(t)} - \mathcal{F}(\boldsymbol{D}\boldsymbol{Z})\right\|_{\mathrm{F}}^2$$

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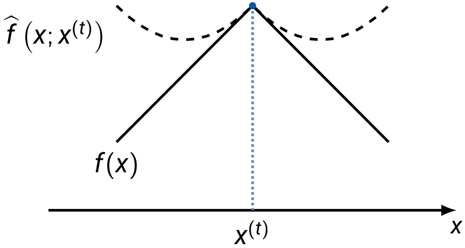

- Limitation: Differential consistency restricts the majorizer at a nondifferentiable point to be nondifferentiable

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Smoothing Majorization
Contribution 1

Restriction

Smoothness of Majorizer

- Lower computational complexity
- Possibility of approximately minimizing the majorizer with SCA

Relaxation

Almost "harmless" relaxation of the convergence set

– Convergence to a stationary point in a relaxed sense

+ Many nondifferentiable stationary points that are not local minima are excluded from the convergence set

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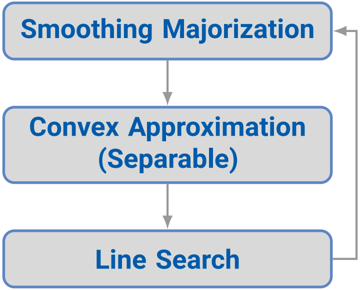
Smoothing Successive Convex Approximation

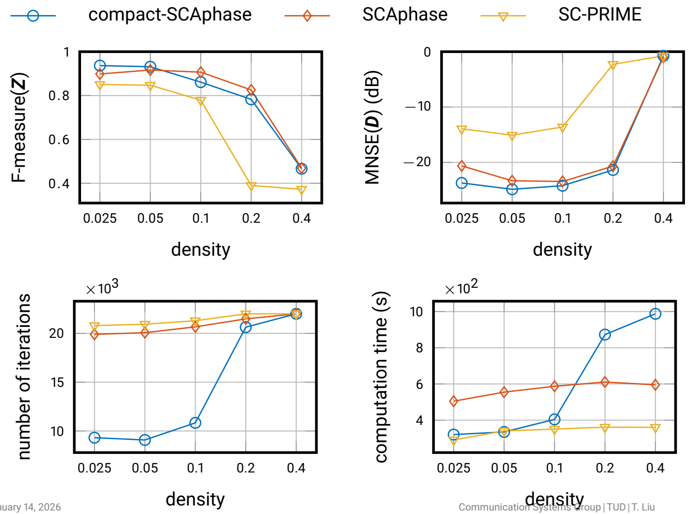
Contribution 2



- The convergence of SCA requires the smoothness of the original function
- Extend SCA to nonsmooth problems with smoothing majorization to enjoy the advantages:
  - Tighter approximation
  - Separable approximation

Simulation Results

Influence of Sparsity Level



- compact-SCAphase
  - Fast convergence in terms of no. of iterations
- SCAphase
  - Low per-iteration complexity
- Benchmark: SC-PRIME [QP17]
  - Block-coordinate-wise majorization

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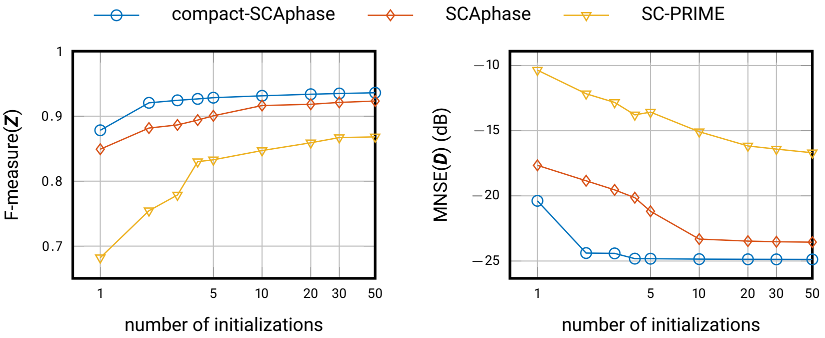
Simulation Results
Robustness to Initialization



- density $= 0.05$
- compact-SCAphase is the most robust to initialization

Smoothing Successive Convex Approximation
Summary of Contributions

- *Smoothing Majorization:* Construct a smooth majorizer for a nonsmooth function
  - Lower computational complexity
  - Relaxed (but insignificantly) convergence set
- *Smoothing SCA:* Extension of SCA to nonsmooth problems with smoothing majorization
- Block coordinate descent version

Outline

Research Works

- Smoothing Successive Convex Approximation
- Gridless DOA Estimation in Distributed Arrays

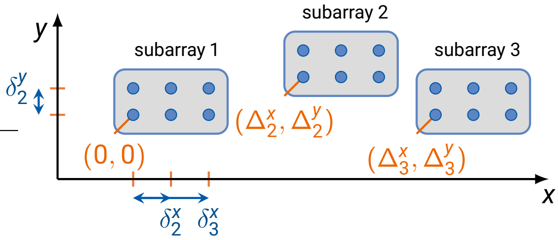
Future Research Directions

Signal Model

Distributed Array with Intersubarray Phase/Displacement Uncertainties

- Identical fully calibrated subarrays

Known	Intrasubarray displacements: δ_l^x, δ_l^y
Unknown	Intersubarray displacements: Δ_p^x, Δ_p^y Phase-offsets: τ_p



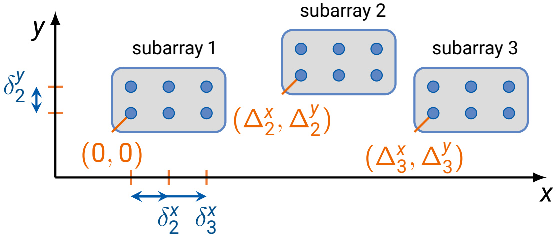
P subarrays of $L_x \times L_y$ sensors

Signal Model

Baseband Measurement over N Snapshots

$$\boldsymbol{Y} = \boldsymbol{A}(\boldsymbol{\mu}, \boldsymbol{\alpha})\boldsymbol{\Psi} + \boldsymbol{N}$$

- $M = P \times L_x \times L_y$: Total number of sensors
- $\boldsymbol{Y} \in \mathbb{C}^{M \times N}$: Received signal matrix
- $\boldsymbol{\Psi} \in \mathbb{C}^{N_\mathrm{S} \times N}$: Source signal matrix
- $\boldsymbol{N} \in \mathbb{C}^{M \times N}$: Sensor noise matrix



P subarrays of $L_x \times L_y$ sensors

- $\boldsymbol{A}(\boldsymbol{\mu}, \boldsymbol{\alpha}) = [\boldsymbol{a}(\mu_1^x, \mu_1^y, \boldsymbol{\alpha}), \ldots, \boldsymbol{a}(\mu_{N_\mathrm{S}}^x, \mu_{N_\mathrm{S}}^y, \boldsymbol{\alpha})] \in \mathbb{C}^{M \times N_\mathrm{S}}$: Steering matrix
- Distinct DOAs from N_S far-field narrowband sources with spatial frequencies (μ_i^x, μ_i^y)
- $\boldsymbol{\mu} = [\mu_1^x, \ldots, \mu_{N_\mathrm{S}}^x, \mu_1^y, \ldots, \mu_{N_\mathrm{S}}^y]^\mathrm{T}$ and $\boldsymbol{\alpha}$ contains all unknown array parameters

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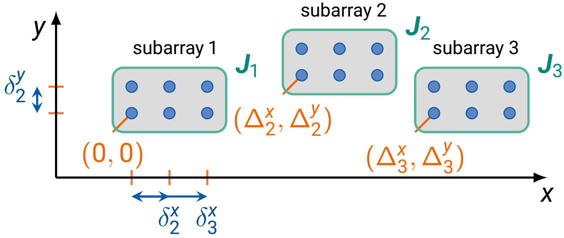
Shift Invariances

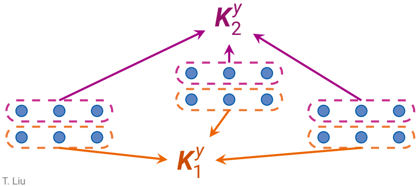
Shift subarrays:

$$\boldsymbol{J}_p{}^{\mathrm{T}}\boldsymbol{A}(\boldsymbol{\mu}) = \boldsymbol{J}_1{}^{\mathrm{T}}\boldsymbol{A}(\boldsymbol{\mu})\boldsymbol{\Phi}(\Delta_p^x\boldsymbol{\mu}^x + \Delta_p^y\boldsymbol{\mu}^y + \tau_p\mathbf{1}),$$
$$p = 2, \ldots, P$$



Shift sensors within a subarray:

$$(\boldsymbol{K}_l^x)^{\mathrm{T}}\boldsymbol{A}(\boldsymbol{\mu}) = (\boldsymbol{K}_1^x)^{\mathrm{T}}\boldsymbol{A}(\boldsymbol{\mu})\boldsymbol{\Phi}(\delta_l^x\boldsymbol{\mu}^x), \quad l = 2, \ldots, L_x$$
$$(\boldsymbol{K}_l^y)^{\mathrm{T}}\boldsymbol{A}(\boldsymbol{\mu}) = (\boldsymbol{K}_1^y)^{\mathrm{T}}\boldsymbol{A}(\boldsymbol{\mu})\boldsymbol{\Phi}(\delta_l^y\boldsymbol{\mu}^y), \quad l = 2, \ldots, L_y$$



- $\boldsymbol{\mu}^x = [\mu_1^x, \ldots, \mu_{N_S}^x]^{\mathrm{T}}$, $\boldsymbol{\mu}^y = [\mu_1^y, \ldots, \mu_{N_S}^y]^{\mathrm{T}}$
- $\boldsymbol{\Phi}(\boldsymbol{x}) = \mathrm{Diag}(e^{jx_1}, \ldots, e^{jx_N}) \in \mathbb{C}^{N\times N}$ for $\boldsymbol{x} \in \mathbb{R}^N$

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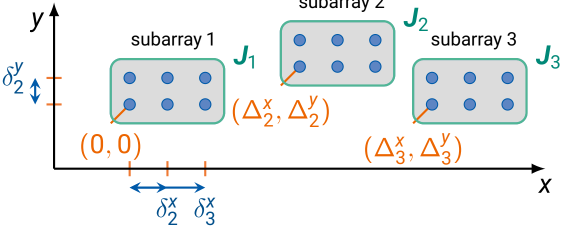
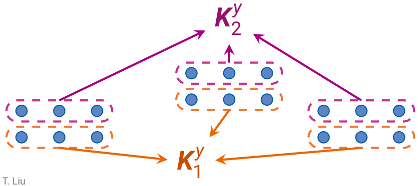


Conventional Approach

ESPRIT-like methods performed on sample covariance matrix [HN98]



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Grid-based Sparse Formulation

- Sparse signal model

$$\boldsymbol{Y} = \boldsymbol{A}(\boldsymbol{\nu}, \boldsymbol{\alpha})\boldsymbol{X} + \boldsymbol{N}$$

  - $\boldsymbol{A}(\boldsymbol{\nu}, \boldsymbol{\alpha}) \in \mathbb{C}^{M \times K}$: Steering matrix for sampled K directions $\boldsymbol{\nu}$ with $K \gg N_\mathrm{S}$
  - $\boldsymbol{X} \in \mathbb{C}^{K \times N}$: Row-sparse representation of $\boldsymbol{\Psi}$

- $\ell_{2,1}$-mixed-norm minimization

$$\widehat{\boldsymbol{X}} = \underset{\boldsymbol{X} \in \mathbb{C}^{K \times N}}{\operatorname{argmin}} \quad \frac{1}{2}\|\boldsymbol{Y} - \boldsymbol{A}(\boldsymbol{\nu}, \boldsymbol{\alpha})\boldsymbol{X}\|_\mathrm{F}^2 + \lambda\sqrt{N}\|\boldsymbol{X}\|_{2,1}$$

  - $\|\boldsymbol{X}\|_{2,1} = \sum_{k=1}^{K} \|\boldsymbol{x}_k\|_2$ for $\boldsymbol{X} = [\boldsymbol{x}_1, \ldots, \boldsymbol{x}_K]^\mathrm{T}$
  - $\lambda > 0$: Regularization parameter

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- SPARROW reformulation [SPP18]

$$\widehat{\boldsymbol{S}} = \underset{\boldsymbol{S} \in \mathbb{D}_+^K}{\operatorname{argmin}} \quad \operatorname{tr}\left(\left(\boldsymbol{A}(\boldsymbol{\nu}, \boldsymbol{\alpha})\boldsymbol{S}\boldsymbol{A}(\boldsymbol{\nu}, \boldsymbol{\alpha})^{\mathsf{H}} + \lambda\boldsymbol{I}_M\right)^{-1}\widehat{\boldsymbol{R}}\right) + \operatorname{tr}(\boldsymbol{S})$$

  - $\widehat{\boldsymbol{S}} = \frac{1}{\sqrt{N}} \operatorname{Diag}(\|\widehat{\boldsymbol{x}}_1\|_2, \ldots, \|\widehat{\boldsymbol{x}}_K\|_2)$

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Shift-Invariant SPARROW

Contribution

- Gridless relaxation of SPARROW $\Longrightarrow$ Shift-Invariant SPARROW (SI-SPARROW)

$$\min_{\boldsymbol{S}\in\mathbb{D}_+^N,\,\boldsymbol{A}\in\mathcal{A}^K,\,\boldsymbol{Q}\in\mathbb{S}_+^M} \quad M\,\mathrm{tr}\left((\boldsymbol{Q}+\lambda\boldsymbol{I}_M)^{-1}\widehat{\boldsymbol{R}}\right)+\mathrm{tr}(\boldsymbol{Q})$$

subject to $\boldsymbol{Q}=\boldsymbol{A}\boldsymbol{S}\boldsymbol{A}^{\mathrm{H}}$ → $\boldsymbol{Q}\in\mathcal{T}^M$

Relaxation

The shift-invariant subspace $\mathcal{T}^M$ is the set of $\boldsymbol{Q}\in\mathbb{S}^M$ that satisfies

$$\begin{aligned}
\boldsymbol{J}_p{}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{J}_p &= \boldsymbol{J}_1{}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{J}_1, && p=2,\dots,P\\
(\boldsymbol{K}_l^x)^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{K}_l^x &= (\boldsymbol{K}_1^x)^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{K}_1^x, && l=2,\dots,L_x\\
(\boldsymbol{K}_l^y)^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{K}_l^y &= (\boldsymbol{K}_1^y)^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{K}_1^y, && l=2,\dots,L_y\\
q_{ii} &= q_{11}, && i=2,\dots,M
\end{aligned}$$

  - $\mathcal{A}^K$: Array manifold with K distinct DOAs
- ESPRIT-like methods performed on $\boldsymbol{Q}$ to recover DOAs
- Sources are decorrelated in $\boldsymbol{Q}$ $\Longrightarrow$ Robust to source correlation

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Shift-Invariant SPARROW

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- Gridless relaxation of SPARROW $\Longrightarrow$ Shift-Invariant SPARROW (SI-SPARROW)

$$\min_{\boldsymbol{S}\in\mathbb{D}_+^N,\,\boldsymbol{A}\in\mathcal{A}^K,\,\boldsymbol{Q}\in\mathbb{S}_+^M} \quad M\,\mathrm{tr}\left((\boldsymbol{Q}+\lambda\boldsymbol{I}_M)^{-1}\widehat{\boldsymbol{R}}\right)+\mathrm{tr}(\boldsymbol{Q})$$

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\end{aligned}$$

  - $\mathcal{A}^K$: Array manifold with K distinct DOAs
- Solution approaches:
  - SDP: auxiliary variables; dependent on general-purpose solvers
  - Parallel algorithm based on ADMM and SCA

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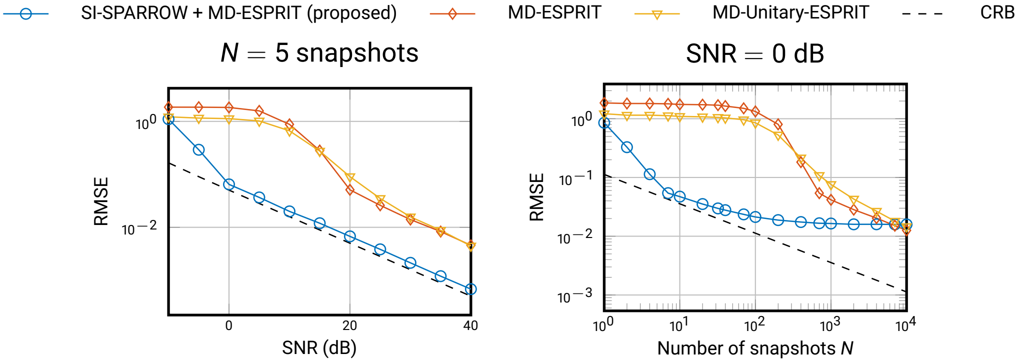
Simulation Results

Robustness to Source Correlation



- 2 sources with correlation coefficient 0.99
- Unitary-ESPRIT [HN98] requires centro-symmetry of the whole array
- SI-SPARROW is robust to source correlation

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Gridless DOA Estimation in Distributed Arrays

Summary of Contributions

- Distributed arrays with inter-subarray phase/displacement uncertainties
- Gridless relaxation of $\ell_{2,1}$-norm minimization based on shift invariances
  - Robustness to source correlation
- Parallel algorithm based on ADMM and SCA

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Outline

Research Works

Future Research Directions

Future Research Directions

Algorithm Unrolling

- Bridge between model-based and data-driven methods
- Hyperparameters have a high impact on performance
- Conventional tuning by cross-validation over training data
- Interpret iterative algorithms as neural networks and learn hyperparameters

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Future Research Directions

Phase Retrieval with Generative Priors

- Signal $\boldsymbol{x}$ is generated by a deep generative model $G(\boldsymbol{z})$ with latent code $\boldsymbol{z}$
- Trained/untrained model G
- Examples:
  - Neural networks
  - Graphical models, e.g., $\boldsymbol{x}$ is generated by an underlying topology with smoothness

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Future Research Directions

Sensing in Heterogeneous Distributed Arrays

- SI-SPARROW: Shift invariances rely on the similarities of subarrays
- Generalized phase retrieval: Partially coherent measurements, e.g., measurements are coherent within each subarray

Thank you for your attention!

References I

[HN98] M. Haardt and J. A. Nossek. "Simultaneous Schur Decomposition of Several Nonsymmetric Matrices to Achieve Automatic Pairing in Multidimensional Harmonic Retrieval Problems". In: *IEEE Trans. Signal Process.* 46.1 (1998), pp. 161–169. ISSN: 1941-0476. DOI: 10.1109/78.651206.

[Liu+19] Tianyi Liu et al. "A Block Coordinate Descent Algorithm for Sparse Gaussian Graphical Model Inference with Laplacian Constraints". In: *IEEE Int. Workshop Comput. Adv. Multi-Sens. Adapt. Process.* Dec. 2019, pp. 236–240. DOI: 10.1109/CAMSAP45676.2019.9022643.

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References II

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[SPP18] Christian Steffens, Marius Pesavento, and Marc E. Pfetsch. “A compact formulation for the $\ell_{2,1}$ mixed-norm minimization problem”. In: *IEEE Trans. Signal Process.* 66.6 (Mar. 2018), pp. 1483–1497. ISSN: 1941-0476. DOI: 10.1109/TSP.2017.2788431.